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APPROXIMATE SOLUTION TO MELTING ICE PROBLEM VIA ADOMIAN DECOMPOSITION METHOD

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ABSTRACT

In this concordance, Adomian Itemization techniques, microwave-prepared to various enliven obstructions in consistence movement in charge to come into ownership of an inexact express arrangement anent respect to Melting Ice issue. The crushing approximations of Adomian Crack-up draw close to going up against assign accordingly go astray the hindrance assets of the moving limit esteem issue are fulfilled. Note the lack of cardinal issuing, the side-effect means are classified and graphically contrasted with these due to a few creators. The numerical front profit oneself of our advantage indicated indubitably acceptable concurrences with others, less disturbed few bases have been adjusted.

KEYWORDS: *Adomian Decomposition Method, Melting Ice Problem, Moving Boundary Value Problem of Partial Differential Equation*

Article History

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1. INTRODUCTION

The term moving limit issues (MBP's) are usually utilized when the limit is related with time subordinate issues and the limit of the area isn't known in cutting edge however must be resolved as an element of time and space. Moving limit issue have gotten much consideration because of their viable significance in designing and science [12]. These issues wind up a nonlinear due present of moving limit [6] and consequently their scientific express arrangement are hard to get when all is said in done.

Stefan issues (stage change issues) is one class of moving limit esteem issue and in addition, application, See Crank[7] and Hill[9]. The class of Stefan issue (MBP'S) is fascinating a direct result of its nonlinearity nature that is related to the moving interface has appeared in [6]. Because of the essence of moving interface, their correct arrangement is restricted. Along these lines, Many rough arrangements have been utilized to take care of this issue numerical [4],[5],[17-20], Stefan issues with time-subordinate limit condition requires some uncommon procedures. In [19], [20], [24]. Savovic and Caldwell [22] exhibited limited distinction arrangement of one-dimensional Stefan issue with occasional limit conditions. Ahmed [3] talked about another calculation for moving limit issue subject to intermittent limit conditions. In 2009, Rajeev et al. [15] utilized variational emphasis strategy to take care of a stage change issue with a time subordinate limit condition and the outcome is gotten in term of Mittag-Leffler work. In 2012 Rajeev and M.S. Kushwaha [14] utilized adomaian disintegration strategy to take care of a Stefan issue with the intermittent limit condition. In 2014

Radhi A.Zaboon and Ahmed I. Mohammed[21] utilized Homotopy Perturbation Method to fathom one – dimensional stage change issue with non – uniform introductory temperature.

In this paper, an inexact unequivocal approach is intrigued by means of an Adomian Decomposition strategy with a few changes the acquired outcomes are contrasted and the correct arrangement [23].

2. DESCRIPTION OF THE PROBLEM (MELTING ICE) *[23]*

The issue of dissolving because of the warmth contribution at a settled limit have been utilized as a part of Furzeland [13] and others, as a moving limit esteem issue (Stefan writes issue) of one stage change issue without introductory condition, and can be available as takes after:

$$\frac{\partial u(x,t)}{\partial t} = \frac{\partial^2 u(x,t)}{\partial x^2}, 0 < x < s(t), \quad 0 < t < 1 \tag{1}$$

Subject to the limit conditions:

$$u_x(0,t) = -e^t, \; t > 0 \tag{2}$$

$$u(s(t),t) = 0, t > 0 \tag{3}$$

$$-\frac{\partial u(s(t),t)}{\partial x} = \frac{ds(t)}{dt}, t > 0 \tag{4}$$

Also, the moving limit is subjected to:

$$s(0) = 0 \tag{5}$$

Where u (x.t) is the temperature at remove x and time t, s(t) being the situation of the interface at time t

3. ANALYSIS OF ADOMIAN DECOMPOSITION METHOD WITH (SIMPLE ALGORITHM)

Think about the condition

$$F(u(x))=g(x) \tag{6}$$

Where F speaks to a general nonlinear customary or fractional differential administrator, including both straight and nonlinear terms, and g is a given capacity. The direct terms in F(u(x)) are deteriorated into Lu+Ru, where L is an effortlessly invertible administrator (more often than not the most noteworthy request subordinate), and R is the rest of the straight administrator. In this way, the condition (6) can be composed as

$$Lu + Ru + Nu = g \tag{7}$$

Where, Nu shows the nonlinear terms. By fathoming this condition (7) for Lu, since L is invertible, and applying the backward administrator L^{-1}on the two sides yields

$$u = A + L^{-1}(g) - L^{-1}(Ru) - L^{-1}(Nu), \tag{8}$$

Where A can be found from the limit or introductory conditions.

Adomian strategy expects the arrangement u can be ventured into infinite series as,

$$u = \sum_{n=0}^{\infty} u_n \tag{9}$$

And $F(u)$ as the summation of a series, say;

$$F(u) = \sum_{n=0}^{\infty} A_n(u_0, u_1, \dots, u_n) \tag{10}$$

WhereA_n's, called Adomian polynomials, has been introduced by the Adomian himself by the formula:

$$A_n(u_0, u_1, \dots, u_n) = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[F\left(\sum_{i=0}^{\infty} u_i \lambda_i\right)\right]_{\lambda=0} \tag{11}$$

Numerous computational algorithms are accessible to process adomian polynomial, for instance [1],[2],[8],[11],[25]. In [10] given an appropriate and more straightforward one, along these lines, we have received this algorithm and as takes after for computing $A_0, A_1, \dots A_n$

Step 1: Input nonlinear term $F(u)$ and n, the number of Adomian polynomial needed.

Step 2: Set $A_0 = F(u_0)$

Step 3: For $k = 0$ to $n - 1$ do:

$$A_k(u_0, u_1, \dots u_k) := A_k(u_o + u_1\lambda, \dots, u_k + (k+1)u_{k+1}\lambda)$$

$$\{in\ A_k : u_i \to u_i + (i+1)u_{i+1}\lambda \quad for\ i = 0,1,\dots,k\ \}$$

Step 4: Taking the first order derivative of A_k, with respect to λ, and then let

$$\lambda = 0: \frac{d}{d\lambda} A_k|_{\lambda=0} = (k+1)A_{k+1}$$

End do

Step 5: Output $A_0, A_1, \dots A_n$.

According to the above Algorithm, Adomian polynomials will be computed as follows:

$$A_0 = F(u_0)$$

$$A_1 = \frac{d}{d\lambda} F(u_0 + u_1\lambda)|_{\lambda=0} = u_1 \acute{F}(u_0),$$

$$A_2 = \frac{1}{2}\frac{d}{d\lambda}\Big((u_1 + 2u_2\lambda)\acute{F}(u_0 + u_1\lambda)\Big)\Big|_{\lambda=0} = u_2\acute{F}(u_0) + \frac{u_1^2}{2!}\acute{\acute{F}}(u_0), \tag{12}$$

And so on. The components of $u_n, n \geq 1$.

Remarks 1

- $u(s(t), t) = 0$, determined the heat distribution at the moving interface equals to zero
- Due to the presence of the moving boundary (1)-(5), the problem is highly nonlinear.
- The initial domain of interest is of length 0, $(0 < x < s(0) = 0)$ at t=0.

4. DETERMINATION OF THE NO MINAL SOLUTION $u_0(x,t)$ AND $s_0(t)$

The nominal solution $u_0(x,t)$ and $s_0(t)$ which are needed for ADM are suggested as follows:

From the Stefan condition (4), we have that

$$\frac{ds(t)}{dt} = -\frac{\partial u(s(t),t)}{\partial x}, \qquad t\epsilon[0,1]$$

That $s(t) \cong s_0(t)$ and at $t = 0$, we have $u(x,t) \cong u_0(x,t)$

$$\left.\frac{ds_0(t)}{dt}\right|_{t=0} = -\left.\frac{\partial u_0(s_0(t),t)}{\partial x}\right|_{t=0}$$

$$\left.\frac{ds_0(t)}{dt}\right|_{t=0} = -\frac{\partial u_0(s_0(0),0)}{\partial x}, \text{from}(5)$$

$$\left.\frac{ds_0(t)}{dt}\right|_{t=0} = -\frac{\partial u_0(0,0)}{\partial x}, \text{from}(2)$$

$\int_0^t \left.\frac{ds_0(t)}{dt}\right|_{t=0} = 1$

$s_0(t) - s_0(0) = t, \text{from}(5)$

$s_0(t) \triangleq t$

$u_0(x,t)$ is selected such that

$$u_0(s_0(t),t) = 0 \ and \ u_{0_x}(0,t) = -e^t$$

Thus, assuming that $u_0(x,t) = \big(x - s_0(t)\big)a$ so that only one condition $u_{0_x}(0,t) = -e^t$ is needed

$u_{0_x}(0,t) = a \triangleq -e^t$

$u_0(x,t) = \big(x - s_0(t)\big)\,(-e^t)$

Hence the nominal solution

$$u_0(x,t) = e^t(s_0(t) - x) \tag{13}$$

$$s_0(t) = t \tag{14}$$

Remarks 2

Based on the problem formulation (1-5) and our choice of the linear operator L of the (ADM), as discussed in (8) the following options are firstly discussed

- On the off chance that one pick L as the whose straight administrators of the issue (1-5) as $L \triangleq L_t - L_{xx}$, and N(u)≜0. The converse administratorL^{-1} is hard to get, subsequently, this choice is discarded.
- If one can choose $L \triangleq L_t$, .The trivial solution is obtained.
- On setting L of (8) as $L \triangleq L_{xx}$ and counting the rest ofL_t in the nonlinear part $N(u)$ of (ADM), this alternative is adjusted and as takes after:

5. SOLUTION OF THE PROBLEM (1-5) VIA (ADM)

Based on remarks (2), write the equation (1) in an operator form

$$L_{xx}u(x,t) = L_t u(x,t), 0 < x < s(t), t > 0 \tag{15}$$

Where$L_{xx} = \frac{\partial^2 .}{\partial x^2}, \ L_t = \frac{\partial .}{\partial t}$

Expecting that the reverse administrator ${L_{xx}}^{-1}$ exists and

$${L_{xx}}^{-1}(.) = \int_0^x \int_0^x (.)dxdx$$

Applying the inverse operator ${L_{xx}}^{-1}$ on both side of the equation (15)

$$u(x,t) - u(0,t) = {L_{xx}}^{-1}\big(L_t u(x,t)\big)$$

Choosing the initial approximation of $u(x,t)$ and $s(t)$ as given in (13),(14)

$$u_0(x,t) = e^t(s_0(t) - x)$$

$$s_0(t) = t$$

According to the Adomian decomposition method (8), decomposition the unknown function $u(x,t)$ as follows:

$$u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + \cdots$$

Where the components $u_0(x,t), u_1(x,t), u_2(x,t)$, ... are defined as

$$u_0(x,t) = u(0,t) = e^t(s_0(t) - x)$$

$$u_1(x,t) = {L_{xx}}^{-1}\big(L_t u_0(x,t)\big) = \int_0^x \int_0^x \Big(\Big(L_t\big(e^t(s_0(t) - x)\big)\Big) dxdx$$

$$= \frac{1}{6}x^2 e^t(3t - x + 3) \qquad \textbf{(16)}$$

$$u_2(x,t) = {L_{xx}}^{-1}\big(L_t u_1(x,t)\big) = \int_0^x \int_0^x \Bigg(\Bigg(L_t\left(\frac{1}{6}x^2 e^t(3t - x + 3)\right)\Bigg) dxdx$$

$$= \frac{1}{120}x^4 e^t(5t - x + 10) \qquad \textbf{(17)}$$

$$u_3(x,t) = {L_{xx}}^{-1}\big(L_t u_2(x,t)\big) = \int_0^x \int_0^x \left(\frac{1}{120}x^4 e^t(5t - x + 10)\right) dxdx$$

$$= \frac{1}{5040}x^6 e^t(7t - x + 21) \qquad \textbf{(18)}$$

$$\vdots$$

Thus

$$u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) + \cdots$$

$$u(x,t) = e^t(s_0(t) - x) + \frac{1}{6}x^2 e^t(3t - x + 3) + \frac{1}{120}x^4 e^t(5t - x + 10) + \frac{1}{5040}x^6 e^t(7t - x + 21) + \cdots \qquad \textbf{(19)}$$

From (4), the Stefan condition for this problem is very interesting in testing the results then on setting:

$$-\frac{\partial u(s(t),t)}{\partial x} = \frac{ds(t)}{dt}$$

On integration both side with respect to *t* from 0 to *t* one gets

$$\int_0^t \frac{ds(t)}{dt} dt = -\int_0^t \frac{\partial u(s(t),t)}{\partial x} dt$$

$$s(t) - s(0) = -\int_0^t \frac{\partial u(s(t),t)}{\partial x} dt\,, s(0) \triangleq s_0(0)$$

$$s(t)s(0) - \int_0^t \frac{\partial u(s(t),t)}{\partial x} dt \tag{20}$$

Decomposing $s(t)$ as,

$$s(t) = \sum_{n=0}^{\infty} s_n(t) \tag{21}$$

When $s_n(t)$ are suitable choose continuously differentiable function based on the nature of the moving $s(t)$ which is changing smoothly on this problem, by this assumption. Using (4) and (19), we have the following:

$$F\big(s(t)\big) = \frac{\partial u(s(t),t)}{\partial x}$$

$$F\big(s_0(t)\big) = \frac{s_0(t)e^t.(3t-s_0(t)+3)}{3} - \frac{\big(s_0(t)\big)^2.e^t}{6} - \frac{\big(s_0(t)\big)^4.e^t}{120} - \frac{\big(s_0(t)\big)^6.e^t}{5040} - e^t + \frac{\big(s_0(t)\big)^3.e^t.(5t-s_0(t)+100)}{30} + \frac{\big(s_0(t)\big)^5.e^t.(7t-s_0(t)+21)}{840} + \cdots \tag{22}$$

Where the initial approximation as assumed $s_0(t) = t$ from (20) and (21), we have got

$$\sum_{n=0}^{\infty} s_n = s_0(0) - \int_0^t \left(\sum_{n=0}^{\infty} A_n\right) dt$$

Where A_n so-called Adomian polynomials for non-linear terms and defined as

$$A_0 = F(s_0),$$

$$A_1 = \frac{dF}{ds_0} s_1,$$

$$A_2 = \frac{dF}{ds_0} s_2 + \frac{1}{2} {s^2}_1 \frac{d^2F}{{ds_0}^2},$$

And so on, the components of $s_n(t), n \geq 1$, can be completely determined as follows:

$$s_1 = \int_0^t A_0\, dt \tag{23}$$

$$s_1 = \int_0^t \left(\frac{s_0(t)e^t.(3t-s_0(t)+3)}{3} - \frac{\big(s_0(t)\big)^2.e^t}{6} - \frac{\big(s_0(t)\big)^4.e^t}{120} - \frac{\big(s_0(t)\big)^6.e^t}{5040} - e^t + \frac{\big(s_0(t)\big)^3.e^t.(5t-s_0(t)+100)}{30} + \frac{\big(s_0(t)\big)^5.e^t.(7t-s_0(t)+21)}{840} + \cdots\right) dt$$

$$s_1 = \left(2e^t + 2t^2e^t - \frac{1}{2}t^3e^t + \frac{5}{24}t^4e^t - \frac{1}{60}t^5e^t + \frac{1}{144}t^6e^t - 3te^t - 2+..\right)$$

And so on. The approximate explicit solution of the moving $s(t)$of the problem (1)-(5) is then obtained by:

$$s(t) = s_0 + s_1 + \cdots$$

$$\mathrm{s(t)} = 0 - \left(2e^t + 2t^2e^t - \frac{1}{2}t^3e^t + \frac{5}{24}t^4e^t - \frac{1}{60}t^5e^t + \frac{1}{144}t^6e^t - 3te^t - 2+..\right) \tag{24}$$

To define an accuracy criterion of this approach, the Stefan condition (4) is used and as follows:

$$\frac{\partial u(s(t),t)}{\partial x} = \frac{te^t.(3t-t+3)}{3} - \frac{(t)^2.e^t}{6} - \frac{(t)^4.e^t}{120} - \frac{(t)^6.e^t}{5040} - e^t + \frac{(t)^3.e^t.(5t-t+100)}{30} + \frac{(t)^5.e^t.(7t-t+21)}{840} + \cdots \tag{25}$$

And

$$\frac{ds(t)}{dt} = e^t - \frac{t^2e^t}{2} - \frac{t^3e^t}{3} - \frac{t^4e^t}{8} - \frac{t^5e^t}{40} - \frac{t^6e^t}{144} - te^t \quad \textbf{(26)}$$

From (25) and (26), the following error criterion is defined and we called it as the absolute error for Stefan condition, i.e

$$\text{Absolute error} \triangleq \left| -\frac{\partial u(s(t),t)}{\partial x} - \frac{ds(t)}{dt} \right| \quad \textbf{(27)}$$

We have used the absolute error (27) and adjusting the number of bases for $u(x,t)$ and $\mathrm{s(t)}$ as follows:

$$u(x,t) = \sum_{i=0}^{n_1} u_i(x,t) \quad \textbf{(28)}$$

$$\mathrm{s(t)} = \sum_{\mathrm{i=0}}^{\mathrm{n_2}} \mathrm{s_i(t)} \quad \textbf{(29)}$$

Based on the following simulation, the number n_1 and $\mathrm{n_2}$ are selected.

The simulation of descritized time-interval $t \in [0,1]$, for sometimes for

$n_1 = 4, n_2 = 2$ is shown below

Table 1

t	$-\frac{\partial u(s(t),t)}{\partial x}$	$\frac{ds(t)}{dt}$	Absolute Error
0	1	1	0
0.01	0.99989916	0.99989882	3.4e-7
0.02	0.99959330	0.99959053	0.00000277
0.03	0.99907734	0.99906781	0.00000953
0.04	0.99834619	0.99832315	0.00002304
0.05	0.99739471	0.99734882	0.00004589
0.06	0.99621776	0.99613685	0.00008091
0.07	0.99481018	0.99467907	0.00013111
0.08	0.99316682	0.99296706	0.00019976
0.09	0.99128254	0.99099216	0.00029038
0.1	0.98915225	0.98874548	0.00040677

Remarks (3)

- Of the table (1) on selection the number n_1 and n_2, we have checked the error for$n_1 = 0,1,2,3$ $n_2 = 0,1$, and then$n_1 = 4, n_2 = 2$ have a reasonable absolute error as show below. Thus $n_1 = 4$, $n_2 = 2$ have been adapted for simplicity. One can also increase the accuracy by selecting more bases in $u(x,t)$ and $s(t)$, i.e $(n_1 > 4, n_2 > 2)$.
- The approximation solution $u(x,t)$ and the moving boundary $s(t)$ are then (13) and(14), for $n_1 = 4$, $n_2 = 2$, respectively
- The comparison have been implemented with exact solution given in [23], where

$$\tilde{u}(x,t) = e^{t-x} - 1 \quad \textbf{(30)}$$

$$\tilde{s}(t) = t \quad \textbf{(31)}$$

From (19) and (24), (30) and (31), the following comparisons are made, where $u(x,t)$ and s(t) are computed using the present approach.

Table 2

t	*exact* $\tilde{s}(t)$	The present method $n_1 = 4, n_2 = 2$ $s(t)$	Absolute of error $\lvert\tilde{s}(t) - s(t)\rvert$
0	0	0	0
0.02	0.02	0.01999728	0.00000272
0.04	0.04	0.03997790	0.0000221
0.06	0.06	0.05992408	0.00007592
0.08	0.08	0.07981682	0.00018318
0.1	0.1	0.09963575	0.00036425

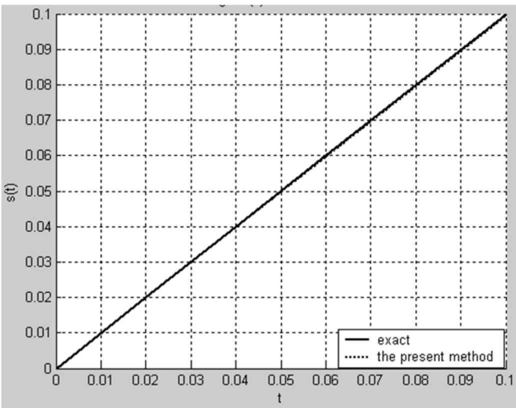

Figure 1

Since the absolute error is the error is very good, even with a very small number of bases, the solution $u(x,t)$ is presented with comparisons for $n_1 = 4$ and as follows

Table 3: The Numerical Results for Different Value of *x* and *t*, with Comparison

t	*x*	Exact $\tilde{u}$ *(x, t)*	$u(x,t)$ by Present Method $n_1 = 4,$	Absolute of Error $\lvert\tilde{u}(x,t) - u(x,t)\rvert$
0	**0**	**0**	**0**	**0**
	0.05	-0.04877057	-0.04877031	2.6e-7
	0.10	-0.09516258	-0.09515841	0.00000417
	0.15	-0.13929202	-0.13927089	0.00002113
	0.20	-0.18126924	-0.18120240	0.00006684
	0.25	-0.22119921	-0.22103577	0.00016344
0.02	**0**	**0.02020134**	**0.02040402**	**0.00020268**
	0.05	-0.02955446	-0.02932600	0.00022846
	0.10	-0.07688365	-0.07657460	0.00030905
	0.15	-0.12190456	-0.12145035	0.00045421
	0.20	-0.16472978	-0.16404946	0.00068032
	0.25	-0.20546639	-0.20445601	0.00101038
0.04	**0**	**0.04081077**	**0.04163243**	**0.00082166**
	0.05	-0.00995016	-0.00907618	0.00087398
	0.10	-0.05823546	-0.05720113	0.00103433
	0.15	-0.10416586	-0.10285297	0.00131289
	0.20	-0.14785621	-0.14612955	0.00172666
	0.25	-0.18941575	-0.18711618	0.00229957
0.06	**0**	**0.06183654**	**0.06371019**	**0.00187365**

Table 3: Contd.,				
	0.05	0.01005016	0.01200374	0.00195358
	0.10	-0.03921056	-0.03701367	0.00219689
	0.15	-0.08606881	-0.08345465	0.00261416
	0.20	-0.13064176	-0.12741868	0.00322308
	0.25	-0.17304086	-0.16899234	0.00404852
0.08	**0**	**0.08328706**	**0.08666296**	**0.0033759**
	0.05	0.03045453	0.03393906	0.00348453
	0.10	-0.01980132	-0.01598723	0.00381409
	0.15	-0.06760618	-0.06323060	0.00437558
	0.20	-0.11307956	-0.10789220	0.00518736
	0.25	-0.15633518	-0.15005988	0.0062753
0.1	**0**	**0.10517091**	**0.11051709**	**0.00534618**
	0.05	0.05127109	0.05675573	0.00548464
	0.10	0.0	0.00590382	0.00590382
	0.15	-0.04877057	-0.04215540	0.00661517
	0.20	-0.09516258	-0.08752481	0.00763777
	0.25	-0.13929202	-0.13029353	0.00899849

Remarks 4

- As appear, the comparison on are very good and shows the efficient of the present approach.
- From table (3) showed the an accuracy is very good, even a small number of a basis for (ADM), $n_1 = 4, n_2 = 2$. To increase the accuracy, one can increase the numbers$n_1$andn_2

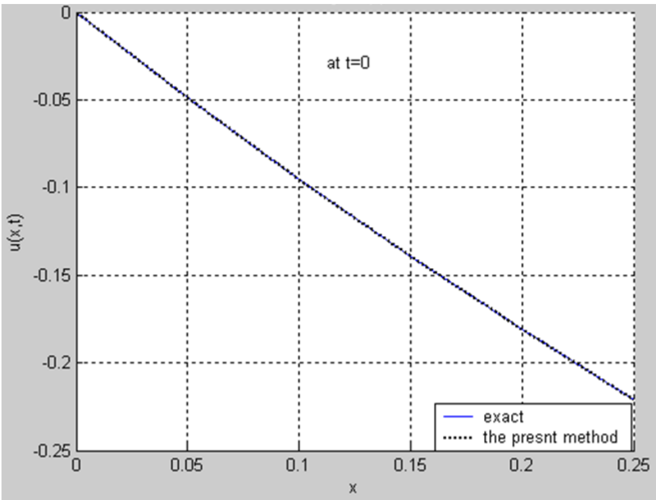

Figure 2

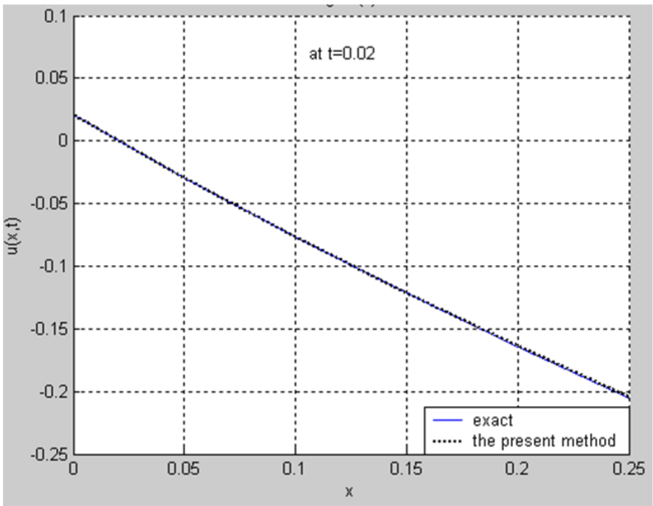

Figure 3

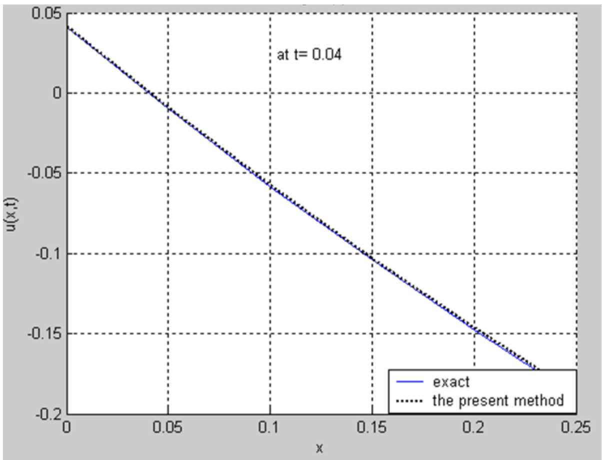

Figure 4

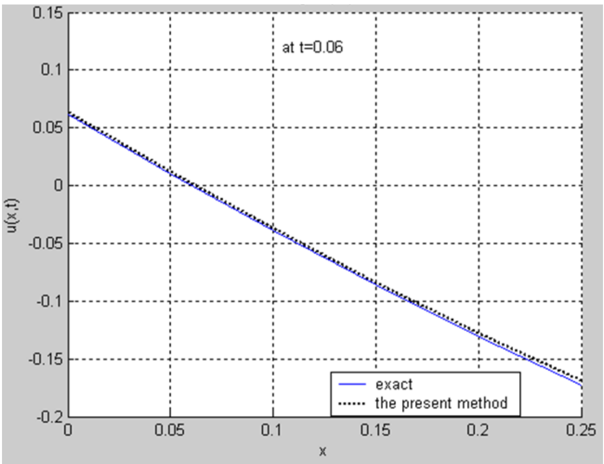

Figure 5

Remarks 5

Figures (2)-(5) present the numerical comparison of $u(x,t)$ and $\tilde{u}(x,t)$ for different value of time

7. CONCLUSIONS

The Adomian Decomposition strategy is effectively connected to locate a surmised unequivocal articulation of temperature dissemination in fluid locale and the interface position of a Stefan issue (1)- (5), respectively, the underlying approximations of u(x, t) and s(t) are selected to accomplish limit state of the first issue (1)- (5) and moving limit condition.

The choice of the ostensible answer for u(x, t) and s(t) by choosing on fitting capacity, fulfilling the limit condition and Stefan condition encourages us to settle such a kind of moving the limit esteem issue effectively and speak to an upgraded approach for such sort of issues.

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